Deductive-Inductive Logic Programming for Collaborative Problem Solving

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Outline

1. Introduction to ILP
2. Integrating Deduction & Induction
3. Collaborative Problem Solving
Deduction Vs. Induction

Deductive Reasoning:
- Examples: Euclidean geometry, Prolog.
- Limitation: does not generalize.

Inductive Reasoning:
- Native to human.
- Central to scientific discovery.
- Limitation: error prone.

Reasoning \{\begin{align*}
\text{Deductive reasoning} & \rightarrow \text{LP} \rightarrow \text{Prolog} \\
\text{Inductive reasoning} & \rightarrow \text{ILP} \rightarrow \text{Aleph}
\end{align*}\}
Inductive Reasoning is classified into:

- **Induction**: Inferring general rules from specific data
  Example: swan example, marble example

- **Abduction**: Reasoning from effects to causes
  Example: medical, detective, scientific

Aim:

- $B \land H \models E$
Inductive Logic Programming
Problem Formulation

Definition (Inductive Logic Programming)

Given $B$ and $E = E^+ \cup E^-$ represented as logic programs, find $H$ such that:

1. Necessity: $B \not\models E^+$
2. Sufficiency: $B \land H \models E^+$
3. Weak Consistency: $B \land H \not\models \Box$
4. Strong Consistency: $B \land H \land E^- \not\models \Box$

How to find $H$ systematically?
Inductive Logic Programming
General Method

General method for finding hypothesis $H$:

**Input:** $B, E^+$ and $E^-$.  
**Output:** $H$.

1. Start with some initial (possibly empty) $H$.
2. **repeat**
3. **if** $B \land H$ is too strong **then**  
4. specialize $H$.  
5. **if** $B \land H$ is too weak **then**  
6. generalize $H$.  
7. **until** all four conditions are met.  
8.  
9. **return** $H$.  

Inductive Logic Programming
Inverse Resolution

Example

\[ B_1 : \text{link}(A,B) \rightarrow \text{reachable}(A,B) \]
\[ B_2 : \text{reachable}(A,B) \land \text{reachable}(B,C) \rightarrow \text{reachable}(A,C) \]
\[ E : \text{reachable}(g,l) \]

\[ H : \text{link}(g,X) \land \text{link}(X,l) \]

\[ H : \text{link}(g,X) \land \text{link}(X,l) \], such that \( B \land H \models E \)
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RichProlog

Limitation of deductive reasoning (e.g. Prolog)
- If A is a swan and is white, B is a swan and is white and C is a swan. What is the color of C?
- If A eats apple, banana, orange, grapes, · · · . Does A eat peach?

Combining deduction and induction
- RichProlog extends Prolog and answers this type of queries: Is there a pattern that matches these instances:

\[(aaa, aab, aba, abb)\]

or

\[\exists X \forall Y \text{pattern}(X) \land \text{matches}(X, Y)\]
Deductive-Inductive Logic Programming Framework

Notation

Let \( \Sigma(T) \) represent deduction, \( \Pi(T) \) represent induction

\[
\Sigma_n(T) = \begin{cases} 
T & \text{if } n = 0 \\
\Sigma(\Pi_{n-1}(T)) & \text{if } n > 0 
\end{cases} \\
\Pi_n(T) = \begin{cases} 
T & \text{if } n = 0 \\
\Pi(\Sigma_{n-1}(T)) & \text{if } n > 0 
\end{cases}
\]

The first few terms of \( \Sigma_n(T) \) and \( \Pi_n(T) \) are as follows:

\[
\begin{align*}
\Sigma_0(T) &= \Pi_0(T) = T & \text{– original theory} \\
\Sigma_1(T) &= \Sigma(\Pi_0(T)) = \Sigma(T) & \text{– deduction on } T \\
\Pi_1(T) &= \Pi(\Sigma_0(T)) = \Pi(T) & \text{– induction on } T \\
\Sigma_2(T) &= \Sigma(\Pi_1(T)) = \Sigma(\Pi(T)) & \ldots \\
\Pi_2(T) &= \Pi(\Sigma_1(T)) = \Pi(\Sigma(T)) & \ldots \\
\vdots & & \vdots
\end{align*}
\]
Deductive-Inductive Logic Programming Framework

Transformation Rules

Deductive-Inductive inference is a recursive application of these rules:

1. $\Sigma(T) \Rightarrow \Sigma(\Sigma(T))$ — Prolog
2. $\Sigma(T) \Rightarrow \Sigma(\Pi(T))$ — RichProlog (roughly)
3. $\Pi(T) \Rightarrow \Pi(\Pi(T))$
4. $\Pi(T) \Rightarrow \Pi(\Sigma(T))$
5. $\Pi(T) \Rightarrow \Sigma(\Pi(T))$ — RichProlog (roughly)

(RichProlog answers ONLY $\Sigma(\Pi(T))$ queries.)
A graphical view of the transformation rules:

Rule 1

Rule 2

Rule 3

Rule 4

Rule 5
Given the following knowledge, can we deduce \( \text{reachable}(b, d) \)?

**Example (Reachability)**

\[
\begin{align*}
&f_1: \text{reachable}(a, c) \\
&f_2: \text{reachable}(a, b) \\
&f_3: \text{reachable}(c, d) \\
&T: \text{reachable}(A, B) \land \text{reachable}(B, C) \to \text{reachable}(A, C) \\
&Q: ?\text{reachable}(b, d)
\end{align*}
\]
Deductive-Inductive Logic Programming Framework
An Example
Problem Solving in Multi-Agent Systems

Issues specific to multi-agent environments:

- Knowledge being distributed;
- Exposure of internal knowledge (cost, privacy, trust etc);
- Knowledge being incomplete;
- Pirates example.

Extend deductive-inductive inference to allow interaction.
Distributed path planning

Problem setting

Each agent has the following background knowledge:

**Example (background knowledge)**

\[ B_1 : \text{link}(A,B) \rightarrow \text{reachable}(A,B) \]
\[ B_2 : \text{reachable}(A,B) \land \text{reachable}(B,C) \rightarrow \text{reachable}(A,C) \]

Plus, each agent knows the links it has travelled, e.g.:

**Example (background knowledge)**

- **Car A**
- **Car B**
- **Car C**
Distributed path planning using DILP

Inducing a path

A path can be induced:

\[
\begin{align*}
B_1 & \quad \text{reachable}(a,g) \land \text{link}(g,j) \land \text{link}(j,l) \\
B_n & \quad \text{reachable}(a,g) \land \text{reachable}(g,l) \\
B_2 & \quad \text{reachable}(a,l) \\
\end{align*}
\]

\[E\]
Distributed path planning using DILP

Collaboration

1. **A**: $E = \text{reachable}(a, l)$
2. **A Asks C**: $E = \text{reachable}(a, l)$
3. **C Induces**: $H = \text{reachable}(a, g) \land \text{link}(g, j) \land \text{link}(j, l)$
4. **C Replies**: $H = \text{reachable}(a, g)$
5. **A Deduces**: $K_1 = K_a(K_c(\text{reachable}(g, l)))$
   $K_2 = K_a(\exists i \ K_i(\text{reachable}(a, g)) \rightarrow K_a(\text{reachable}(a, l)))$
6. **A**: $E = \text{reachable}(a, g)$

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**Diagram**:

- **Car A**: Nodes b, c, d, a
- **Car B**: Nodes e, f, g, h, c, d
- **Car C**: Nodes f, j, k, g, l
Distributed path planning using DILP

Collaboration

6 A: \( E = \text{reachable}(a, g) \)

7 A ASKS B: \( E = \text{reachable}(a, g) \)

8 B INDUCES: \( H = \text{reachable}(a, c) \land \text{link}(c, d) \land \text{link}(d, g) \)

9 B REPLIES: \( H = \text{reachable}(a, c) \)

10 A DEDUCES: \( K_3 = K_a(K_b(\text{reachable}(c, g))) \)
\( K_4 = K_a(\exists i K_i(\text{reachable}(a, c)) \rightarrow K_a(\text{reachable}(a, g))) \)

11 A: \( E = \text{reachable}(a, c) \)

12 A INDUCES: \( H = \text{link}(a, c) \)
Experiments to investigate communication cost:

- compare DILP approach against a centralized approach
- different numbers of agents, $A$, from 2 to 6
- varying graph sizes, $G$, from 60 to 120
- 100 trials for every value of $A$ and $G$
1. Communication increases as knowledge is more evenly distributed.
2. Communication increase much slower with DILP approach.
3. Communication increases as number of agents increases.
4. Communication increases as graph size increases.
5. Communication lower with DILP in general, except when each agent knows only very few links.

6. Communication lower with DILP when each agent knows 30 or more links.
Conclusion

1. Deduction Vs. Induction
   - Limitations with deduction and induction
   - ILP methods and techniques

2. Integrating Deductive and Inductive Reasoning
   - RichProlog
   - Deductive-Inductive Logic Programming Framework
   - Transformation Rules

3. Application in Collaborative Problem Solving
   - Collaboration in multi-agent environment
   - Integrates deduction, induction and interaction
   - Path planning example

4. Results show promise for:
   - overcoming the problem of knowledge being distributed;
   - avoiding central control;
   - saving communication cost.